nrefocus Documentation

Release 0.4.0

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CONTENTS:

Nrefocus is a Python 3 library that allows to numerically refocus (including autofocusing) complex wave fields. This is the documentaion of nrefocus version 0.4.0.

CHAPTER

ONE

INTRODUCTION

This package provides methods for numerical propagation of a complex wave in free space. The available propagators are the angular spectrum method (*helmholtz*) and the Fresnel approximation (*fresnel*). Both implementations are convolution-based. The angular spectrum method is suited for near-field propagation (numerical focusing) and yields better results than the Fresnel approximation. The single Fourer transform-based Fresnel propagation method which is suitable for far-field propagation is not implemented in this package.

1.1 Obtaining nrefocus

You can install nrefocus via:

```
pip install nrefocus
```

If you would like to take advantage of fast Fourer transforms with PyFFTW, please also install the *pyfftw* package or use the extras key *FFTW*:

```
pip install nrefocus[FFTW]
```

The source code of nrefocus is available at https://github.com/RI-imaging/nrefocus.

1.2 Citing nrefocus

Please cite this package if you are using it in a scientific publication.

This package should be cited like this[1].

You can find out what version you are using by typing (in a Python console):

```
>>> import nrefocus
>>> nrefocus.__version__
'0.1.2'
```

[1] Paul Müller (2013) *nrefocus: Python algorithms for numerical focusing* (Version x.x.x) [Software]. Available at https://pypi.python.org/pypi/nrefocus/.

1.3 Acknowledgments

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1.4 References

CHAPTER

TWO

THEORY

The derivations given here are treated in more detail in the relevant literature, e.g. [[ST91]] and [[Goo05]].

2.1 Optical transfer function

Let us consider a wave field $u(\mathbf{r_0})$ whose values we know at an initial plane $\mathbf{r_0} = (x_0, y_0, z_0)$ (z_0 fixed). The field has a certain vacuum wavelength λ and is traveling through a homogeneous medium with refractive index n_m. From the knowledge of the wave field at the plane $\mathbf{r_0}$ and its wavelength λ/n_m, we can infer the direction of propagation of the wave field for every point in $\mathbf{r_0}$. We rewrite the field at $\mathbf{r_0}$ as an angular spectrum, a sum over all possible directions $\mathbf{s} = (p, q, M)$, assuming that the field is only traveling from left to right

$$\begin{aligned} u(\mathbf{r_0}) &= \iint dpdq\, A(p,q) e^{ik_\mathrm{m}(px_0 + qy_0 + Mz_0)} \\ |\mathbf{s}| &= p^2 + q^2 + M^2 = 1 \\ M &= \sqrt{1 - p^2 - q^2}. \end{aligned}$$

The equation above describes the Huygens-Fresnel principle: the value of the field u at a certain position $\mathbf{r_0}$ at the initial plane (point source) is defined as an integral over all possible plane waves with wavenumber $k_\mathrm{m} = \frac{2\pi n_\mathrm{m}}{\lambda}$, weighted with the amplitude $A(p, q)$.

Let us now consider the 2D Fourier transform of $u(\mathbf{r_0})$.

$$\begin{aligned} \widehat{U}_0(k_\mathrm{x}, k_\mathrm{y}) &= \frac{1}{2\pi} \iint dx_0 dy_0 \iint dpdq\, A(p,q) e^{ik_\mathrm{m}(px_0 + qy_0 + Mz_0)} e^{-i(k_\mathrm{x}x_0 + k_\mathrm{y}y_0)} \\ &= \frac{1}{2\pi} \iint dx_0 dy_0 \iint dpdq\, A(p,q) e^{ik_\mathrm{m}Mz_0} e^{ix_0(k_\mathrm{m}p - k_\mathrm{x})} e^{iy_0(k_\mathrm{m}q - k_\mathrm{y})} \\ &= \frac{2\pi}{k_\mathrm{m}^2} A(k_\mathrm{x}, k_\mathrm{y}) e^{ik_\mathrm{m}Mz_0} \end{aligned}$$

Here we made use of the identity of the delta distribution

$$\begin{aligned} \frac{1}{2\pi} \int dx_0\, e^{ix_0(k_\mathrm{m}p - k_\mathrm{x})} &= \delta(k_\mathrm{m}p - k_\mathrm{x}) = \frac{1}{k_\mathrm{m}} \delta(p - k_\mathrm{x}/k_\mathrm{m}) \\ \frac{1}{2\pi} \int dy_0\, e^{iy_0(k_\mathrm{m}q - k_\mathrm{y})} &= \delta(k_\mathrm{m}q - k_\mathrm{y}) = \frac{1}{k_\mathrm{m}} \delta(q - k_\mathrm{y}/k_\mathrm{m}) \end{aligned}$$

If we now perform the same procedure for a different position $\mathbf{r}_\mathrm{d} = (x_0, y_0, z_\mathrm{d})$, we will see that the Fourier transform of the field becomes

$$\widehat{U}_\mathrm{d}(k_\mathrm{x}, k_\mathrm{y}) = \frac{2\pi}{k_\mathrm{m}^2} A(k_\mathrm{x}, k_\mathrm{y}) e^{ik_\mathrm{m}Mz_\mathrm{d}}.$$

Thus, the propagation of the field $u(\mathbf{r_0})$ by a distance $d = z_\mathrm{d} - z_0$ is described by a multiplication with the transfer function

$$\mathcal{H}^\mathrm{Helmholtz} = e^{ik_\mathrm{m} M d}$$

in Fourier space. This is the basis of the convolution-based numerical propagation algorithms implemented in nrefocus. The process of numerical propagation with the angular spectrum method can be written as

$$u(\mathbf{r_d}) = \mathcal{F}^{-1}\left\{ \mathcal{F}\{u(\mathbf{r_0})\} \cdot e^{ik_\mathrm{m} M d} \right\}$$

with the Fourier transform $\mathcal{F}$ and its inverse $\mathcal{F}^{-1}$. With the convolution operator $*$, we may rewrite this equation to

$$u(\mathbf{r_d}) = u(\mathbf{r_0}) * \mathcal{F}^{-1}\left\{ e^{ik_\mathrm{m} M d} \right\}.$$

2.2 Fresnel approximation

The Fresnel approximation (or paraxial approximation) uses a Taylor expansion to simplify the exponent of the transfer function $e^{ik_\mathrm{m} M d}$. The exponent can be rewritten as

$$ik_\mathrm{m} M d = ik_\mathrm{m} d \left(1 - p^2 - q^2\right)^{1/2}.$$

If the angles of propagation θ_x and θ_y for each plane wave of the angular spectrum is small, then we can make the paraxial approximation:

$$\begin{aligned} \theta_\mathrm{x} &\approx p \\ \theta_\mathrm{y} &\approx q \\ \theta^2 = \theta_\mathrm{x}^2 + \theta_\mathrm{y}^2 &\approx p^2 + q^2 \end{aligned}$$

We now Taylor-expand the exponent around small values of θ

$$ik_\mathrm{m} d \left(1 - \theta^2\right)^{1/2} \approx ik_\mathrm{m} d \left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{8} - \ldots\right).$$

The Fresnel approximation discards the third term ($\sim \theta^4$) and the transfer function then reads:

$$\begin{aligned} e^{ik_\mathrm{m} M d} &\approx e^{ik_\mathrm{m} d} \cdot e^{-\frac{ik_\mathrm{m} d (p^2+q^2)}{2}} \\ e^{i\sqrt{k_\mathrm{m}^2 - k_\mathrm{x}^2 - k_\mathrm{y}^2} d} &\approx e^{ik_\mathrm{m} d} \cdot e^{-\frac{id(k_\mathrm{x}^2+k_\mathrm{y}^2)}{2k_\mathrm{m}}} \\ \mathcal{H}^\mathrm{Fresnel} &= e^{ik_\mathrm{m} d} \cdot e^{-\frac{id(k_\mathrm{x}^2+k_\mathrm{y}^2)}{2k_\mathrm{m}}} \end{aligned}$$

Thus, the propagation by a distance distance $d = z_\mathrm{d} - d$ in the Fresnel approximation can be written in the form of the convolution

$$u(\mathbf{r_d}) = e^{ik_\mathrm{m} d} \cdot u(\mathbf{r_0}) * \mathcal{F}^{-1}\left\{ e^{-\frac{id(k_\mathrm{x}^2+k_\mathrm{y}^2)}{2k_\mathrm{m}}} \right\}.$$

Note that the Fresnel approximation results in paraboloidal waves $(p^2 + q^2)$ whereas spherical waves are used with the Helmholtz equation.

2.3 Transfer functions in nrefocus

The numerical focusing algorithms in this package require the input data u_{in} to be normalized by the incident plane wave $u_0(\mathbf{r_0})$ according to

$$u_{\text{in}}(\mathbf{r_0}) = \frac{u(\mathbf{r_0})}{u_0(\mathbf{r_0})}$$

As a result, the transfer functions change to

$$\begin{aligned}\mathcal{H}_{\text{norm}}^{\text{Helmholtz}} &= e^{ik_\mathrm{m}(M-1)d} = e^{id\left(\sqrt{k_\mathrm{m}^2 - k_\mathrm{x}^2 - k_\mathrm{y}^2} - k_\mathrm{m}\right)} \\ \mathcal{H}_{\text{norm}}^{\text{Fresnel}} &= e^{-\frac{id(k_\mathrm{x}^2 + k_\mathrm{y}^2)}{2k_\mathrm{m}}}.\end{aligned}$$

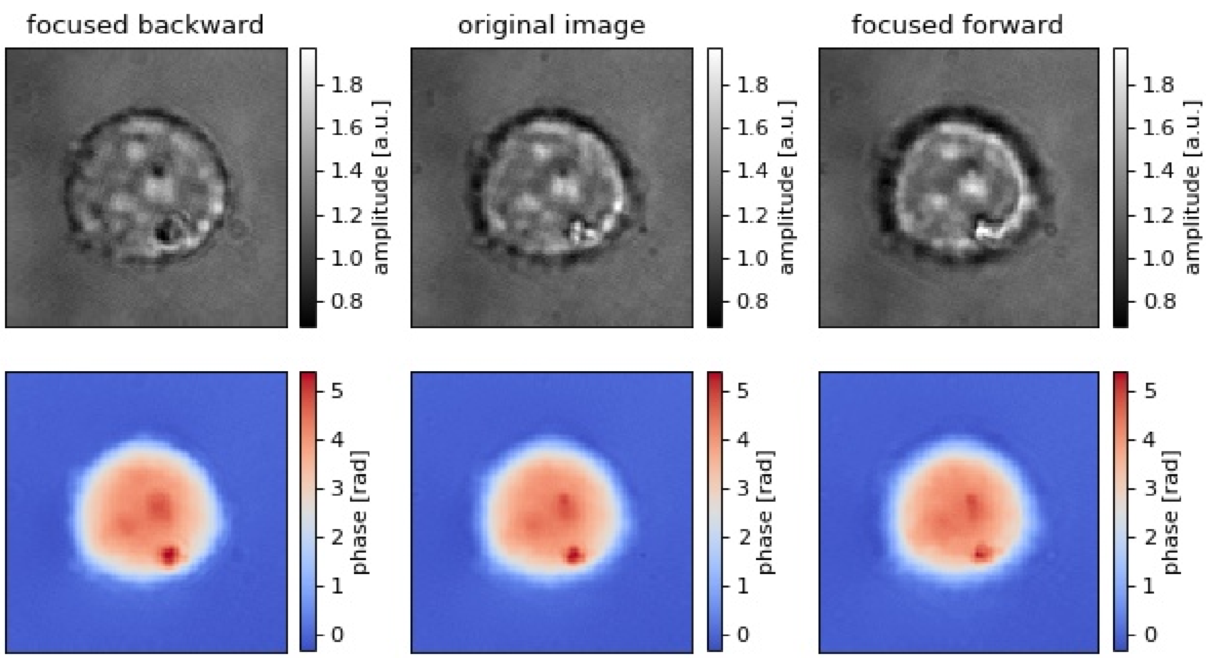
CHAPTER

THREE

EXAMPLES

3.1 2D Refocusing of an HL60 cell

The data show a live HL60 cell imaged with quadriwave lateral shearing interferometry (SID4Bio, Phasics S.A., France). The diameter of the cell is about 20µm.



refocus_cell.py

```python
import matplotlib.pylab as plt
import numpy as np
import unwrap

import nrefocus

from example_helper import load_cell

# load initial cell
cell1 = load_cell("HL60_field.zip")
```

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```python

# refocus to two different positions
cell2 = nrefocus.refocus(cell1, 15, 1, 1)  # forward
cell3 = nrefocus.refocus(cell1, -15, 1, 1)  # backward

# amplitude range
vmina = np.min(np.abs(cell1))
vmaxa = np.max(np.abs(cell1))
ampkw = {"cmap": plt.get_cmap("gray"),
         "vmin": vmina,
         "vmax": vmaxa}

# phase range
cell1p = unwrap.unwrap(np.angle(cell1))
cell2p = unwrap.unwrap(np.angle(cell2))
cell3p = unwrap.unwrap(np.angle(cell3))
vminp = np.min(cell1p)
vmaxp = np.max(cell1p)
phakw = {"cmap": plt.get_cmap("coolwarm"),
         "vmin": vminp,
         "vmax": vmaxp}

# plots
fig, axes = plt.subplots(2, 3, figsize=(8, 4.5))
axes = axes.flatten()
for ax in axes:
    ax.xaxis.set_major_locator(plt.NullLocator())
    ax.yaxis.set_major_locator(plt.NullLocator())

# titles
axes[0].set_title("focused backward")
axes[1].set_title("original image")
axes[2].set_title("focused forward")

# data
mapamp = axes[0].imshow(np.abs(cell3), **ampkw)
axes[1].imshow(np.abs(cell1), **ampkw)
axes[2].imshow(np.abs(cell2), **ampkw)
mappha = axes[3].imshow(cell3p, **phakw)
axes[4].imshow(cell1p, **phakw)
axes[5].imshow(cell2p, **phakw)

# colobars
cbkwargs = {"fraction": 0.045}
plt.colorbar(mapamp, ax=axes[0], label="amplitude [a.u.]", **cbkwargs)
plt.colorbar(mapamp, ax=axes[1], label="amplitude [a.u.]", **cbkwargs)
plt.colorbar(mapamp, ax=axes[2], label="amplitude [a.u.]", **cbkwargs)
plt.colorbar(mappha, ax=axes[3], label="phase [rad]", **cbkwargs)
plt.colorbar(mappha, ax=axes[4], label="phase [rad]", **cbkwargs)
plt.colorbar(mappha, ax=axes[5], label="phase [rad]", **cbkwargs)

plt.tight_layout()
plt.show()
```

CHAPTER

FOUR

CODE REFERENCE

4.1 Refocus interface

Refocus is a user-convenient interface for numerical refocusing. Each class implements refocusing for a specific dimensionality (1D or 2D fields) using a specific method for refocusing (e.g. numpy FFT or FFTW).

4.2 Metrics

4.3 Legacy methods

These methods are legacy functions which are kept for backwards-compatibility.

4.3.1 Refocusing

4.3.2 Autofocusing

CHAPTER

FIVE

CHANGELOG

List of changes in-between nrefocus releases.

5.1 version 0.4.0

- feat: implement nrefocus.RefocusPyFFTW for faster refocusing using pyfftw
- enh: speed-up propagation kernel computation using numexpr
- docs: cleanup

5.2 version 0.3.1

- dist: include submodules in wheel/dist

5.3 version 0.3.0

- feat: introduce nrefocus.RefocusNumpy and nrefocus.RefocusNumpy1D interface class for user-convenience and efficiency
- docs: cleanup
- ref: new submodule for metrics and metrics now accept a Refocus instance as an argument
- ref: new submodule for minimizers and minimizers now accept a Refocus instance
- ref: make legacy autofocusing code use the new Refocus class

5.4 version 0.2.1

- fix: fix several minor bugs (deprecations?) that caused the tests to faile
- ci: migrate to GitHub Actions
- setup: setup.py test is deprecated
- docs: refurbish documentation

5.5 version 0.2.0

- Drop support for Python 2 (#8)
- Code cleanup

5.6 version 0.1.8

- Include docs in sdist

5.7 version 0.1.7

- Update documentation and examples

5.8 version 0.1.6

- Move documentation from GitHub to readthedocs.io
- Add universal wheel on PyPI
- Update tests on travis with new versions of NumPy

5.9 version 0.1.5

- Code cleanup

5.10 version 0.1.4

- Padding is now available in all methods (#2)
- Added new convenient submodule *pad*
- Bugfix: autofocusing did not return the correct focusing distance. This resulted in a slight offset in the refocusing distance for the method *autofocus_stack* when *same_dist=True* was set.
- New test functions for *pad*

CHAPTER

SIX

BILBLIOGRAPHY

CHAPTER

SEVEN

INDICES AND TABLES

- genindex
- modindex
- search

BIBLIOGRAPHY

[Goo05] Joseph W. Goodman. *Introduction to Fourier Optics 3rd ed*. Roberts & Company Publishers, 2005.

[ST91] Bahaa E. A. Saleh and Malvin Carl Teich. *Fundamentals of Photonics*. John Wiley & Sons, Inc., aug 1991. doi:10.1002/0471213748.