
nrefocus Documentation

Release 0.3.1

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Nrefocus is a Python 3 library that allows to numerically refocus (including autofocusing) complex wave fields. This is the documentaion of nrefocus version 0.3.1.

INTRODUCTION

This package provides methods for numerical propagation of a complex wave in free space. The available propagators are the angular spectrum method (*helmholtz*) and the Fresnel approximation (*fresnel*). Both implementations are convolution-based. The angular spectrum method is suited for near-field propagation (numerical focusing) and yields better results than the Fresnel approximation. The single Fourier transform-based Fresnel propagation method which is suitable for far-field propagation is not implemented in this package.

1.1 Obtaining nrefocus

If you have Python and `numpy` installed, simply run

```
pip install nrefocus
```

The source code of nrefocus is available at <https://github.com/RI-imaging/nrefocus>.

1.2 Citing nrefocus

Please cite this package if you are using it in a scientific publication.

This package should be cited like this¹.

You can find out what version you are using by typing (in a Python console):

```
>>> import nrefocus
>>> nrefocus.__version__
'0.1.2'
```

1.3 Acknowledgments

This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement no 282060.

¹ Paul Müller (2013) *nrefocus: Python algorithms for numerical focusing* (Version x.x.x) [Software]. Available at <https://pypi.python.org/pypi/nrefocus/>.

1.4 References

THEORY

The derivations given here are treated in more detail in the relevant literature, e.g. [[ST91]] and [[Goo05]].

2.1 Optical transfer function

Let us consider a wave field $u(\mathbf{r}_0)$ whose values we know at an initial plane $\mathbf{r}_0 = (x_0, y_0, z_0)$ (z_0 fixed). The field has a certain vacuum wavelength λ and is traveling through a homogeneous medium with refractive index n_m . From the knowledge of the wave field at the plane \mathbf{r}_0 and its wavelength λ/n_m , we can infer the direction of propagation of the wave field for every point in \mathbf{r}_0 . We rewrite the field at \mathbf{r}_0 as an angular spectrum, a sum over all possible directions $\mathbf{s} = (p, q, M)$, assuming that the field is only traveling from left to right

$$\begin{aligned} u(\mathbf{r}_0) &= \iint dp dq A(p, q) e^{ik_m(px_0 + qy_0 + Mz_0)} \\ |\mathbf{s}| &= p^2 + q^2 + M^2 = 1 \\ M &= \sqrt{1 - p^2 - q^2}. \end{aligned}$$

The equation above describes the Huygens-Fresnel principle: the value of the field u at a certain position \mathbf{r}_0 at the initial plane (point source) is defined as an integral over all possible plane waves with wavenumber $k_m = \frac{2\pi n_m}{\lambda}$, weighted with the amplitude $A(p, q)$.

Let us now consider the 2D Fourier transform of $u(\mathbf{r}_0)$.

$$\begin{aligned} \hat{U}_0(k_x, k_y) &= \frac{1}{2\pi} \iint dx_0 dy_0 \iint dp dq A(p, q) e^{ik_m(px_0 + qy_0 + Mz_0)} e^{-i(k_x x_0 + k_y y_0)} \\ &= \frac{1}{2\pi} \iint dx_0 dy_0 \iint dp dq A(p, q) e^{ik_m M z_0} e^{ix_0(k_m p - k_x)} e^{iy_0(k_m q - k_y)} \\ &= \frac{2\pi}{k_m^2} A(k_x, k_y) e^{ik_m M z_0} \end{aligned}$$

Here we made use of the identity of the delta distribution

$$\begin{aligned} \frac{1}{2\pi} \int dx_0 e^{ix_0(k_m p - k_x)} &= \delta(k_m p - k_x) = \frac{1}{k_m} \delta(p - k_x/k_m) \\ \frac{1}{2\pi} \int dy_0 e^{iy_0(k_m q - k_y)} &= \delta(k_m q - k_y) = \frac{1}{k_m} \delta(q - k_y/k_m) \end{aligned}$$

If we now perform the same procedure for a different position $\mathbf{r}_d = (x_0, y_0, z_d)$, we will see that the Fourier transform of the field becomes

$$\hat{U}_d(k_x, k_y) = \frac{2\pi}{k_m^2} A(k_x, k_y) e^{ik_m M z_d}.$$

Thus, the propagation of the field $u(\mathbf{r}_0)$ by a distance $d = z_d - z_0$ is described by a multiplication with the transfer function

$$\mathcal{H}^{\text{Helmholtz}} = e^{ik_m M d}$$

in Fourier space. This is the basis of the convolution-based numerical propagation algorithms implemented in nrefocus. The process of numerical propagation with the angular spectrum method can be written as

$$u(\mathbf{r}_d) = \mathcal{F}^{-1} \{ \mathcal{F} \{ u(\mathbf{r}_0) \} \cdot e^{ik_m M d} \}$$

with the Fourier transform \mathcal{F} and its inverse \mathcal{F}^{-1} . With the convolution operator $*$, we may rewrite this equation to

$$u(\mathbf{r}_d) = u(\mathbf{r}_0) * \mathcal{F}^{-1} \{ e^{ik_m M d} \}.$$

2.2 Fresnel approximation

The Fresnel approximation (or paraxial approximation) uses a Taylor expansion to simplify the exponent of the transfer function $e^{ik_m M d}$. The exponent can be rewritten as

$$ik_m M d = ik_m d (1 - p^2 - q^2)^{1/2}.$$

If the angles of propagation θ_x and θ_y for each plane wave of the angular spectrum is small, then we can make the paraxial approximation:

$$\begin{aligned} \theta_x &\approx p \\ \theta_y &\approx q \\ \theta^2 &= \theta_x^2 + \theta_y^2 \approx p^2 + q^2 \end{aligned}$$

We now Taylor-expand the exponent around small values of θ

$$ik_m d (1 - \theta^2)^{1/2} \approx ik_m d \left(1 - \frac{\theta^2}{2} + \frac{\theta^4}{8} - \dots \right).$$

The Fresnel approximation discards the third term ($\sim \theta^4$) and the transfer function then reads:

$$\begin{aligned} e^{ik_m M d} &\approx e^{ik_m d} \cdot e^{-\frac{ik_m d(p^2 + q^2)}{2}} \\ e^{i\sqrt{k_m^2 - k_x^2 - k_y^2} d} &\approx e^{ik_m d} \cdot e^{-\frac{id(k_x^2 + k_y^2)}{2k_m}} \\ \mathcal{H}^{\text{Fresnel}} &= e^{ik_m d} \cdot e^{-\frac{id(k_x^2 + k_y^2)}{2k_m}} \end{aligned}$$

Thus, the propagation by a distance $d = z_d - d$ in the Fresnel approximation can be written in the form of the convolution

$$u(\mathbf{r}_d) = e^{ik_m d} \cdot u(\mathbf{r}_0) * \mathcal{F}^{-1} \left\{ e^{-\frac{id(k_x^2 + k_y^2)}{2k_m}} \right\}.$$

Note that the Fresnel approximation results in paraboloidal waves ($p^2 + q^2$) whereas spherical waves are used with the Helmholtz equation.

2.3 Transfer functions in nrefocus

The numerical focusing algorithms in this package require the input data u_{in} to be normalized by the incident plane wave $u_0(\mathbf{r}_0)$ according to

$$u_{\text{in}}(\mathbf{r}_0) = \frac{u(\mathbf{r}_0)}{u_0(\mathbf{r}_0)}$$

As a result, the transfer functions change to

$$\begin{aligned}\mathcal{H}_{\text{norm}}^{\text{Helmholtz}} &= e^{ik_{\text{m}}(M-1)d} = e^{id(\sqrt{k_{\text{m}}^2 - k_{\text{x}}^2 - k_{\text{y}}^2} - k_{\text{m}})} \\ \mathcal{H}_{\text{norm}}^{\text{Fresnel}} &= e^{-\frac{id(k_{\text{x}}^2 + k_{\text{y}}^2)}{2k_{\text{m}}}}.\end{aligned}$$

- **roi** (*rectangular region of interest (x1, y1, x2, y2)*) – Region of interest of *field* for which the metric will be minimized. If not given, the entire *field* will be used.

Returns

- **af_field** (*2d ndarray*) – Autofocused field
- **af_distance** (*float*) – Autofocusing distance

get_kernel (*distance*)

Return the current kernel

The kernel type *self.kernel* is used (see `Refocus.__init__()`)**propagate** (*distance*)

Propagate the initial field to a certain distance

Parameters **distance** (*float*) – Absolute focusing distance [m]**Returns** **refocused_field** – Initial field refocused at *distance***Return type** 2d ndarray**Notes**Any subclass should perform padding with `nrefocus.pad.pad_rem()` during initialization.**property shape**

Shape of the padded input field or Fourier transform

class `nrefocus.RefocusNumpy1D` (*field, wavelength, pixel_size, medium_index=1.3333, distance=0, kernel='helmholtz', padding=True*)

Refocus a 1D field with numpy

Parameters

- **field** (*1d complex-valued ndarray*) – Input 1D field to be refocused
- **wavelength** (*float*) – Wavelength of the used light [m]
- **pixel_size** (*float*) – Pixel size of the input image [m]
- **medium_index** (*float*) – Refractive index of the medium, defaults to water (1.3333 at 21.5°C)
- **distance** (*float*) – Initial focusing distance [m]
- **kernel** (*str*) – Propagation kernel, one of
 - "helmholtz": the optical transfer function $\exp\left(id\left(\sqrt{k_m^2 - k_x^2} - k_m\right)\right)$
 - "fresnel": paraxial approximation $\exp(-idk_x^2/2k_m)$
- **padding** (*bool*) – Whether or not to perform zero-padding

autofocus (*metric='average gradient', minimizer='legacy', interval=(None, None), roi=None, minimizer_kwargs=None*)

Autofocus the initial field

Parameters

- **metric** (*str*) –
 - "average gradient": average gradient metric of amplitude

- "rms contrast" : RMS contrast of phase data
- "spectrum" : sum of filtered Fourier coefficients
- **minimizer** (*str*) -
 - "legacy": custom nrefocus minimizer
- **interval** (*tuple of floats*) - Approximate interval to search for optimal focus in px.
- **roi** (*rectangular region of interest (x1, y1, x2, y2)*) - Region of interest of *field* for which the metric will be minimized. If not given, the entire *field* will be used.

Returns

- **af_field** (*2d ndarray*) - Autofocused field
- **af_distance** (*float*) - Autofocusing distance

get_kernel (*distance*)

Return the kernel for a 1D propagation

propagate (*distance*)

Propagate the initial field to a certain distance

Parameters **distance** (*float*) - Absolute focusing distance [m]

Returns **refocused_field** - Initial 1D field refocused at *distance*

Return type 1d ndarray

property shape

Shape of the padded input field or Fourier transform

3.2 Metrics

`nrefocus.metrics.metric_average_gradient` (*rfi, distance, roi=None, *kwargs*)

Compute mean average gradient norm of the amplitude

Notes

The absolute value of the gradient is returned.

`nrefocus.metrics.metric_rms_contrast` (*rfi, distance, roi=None, *kwargs*)

Compute RMS contrast of the phase

Notes

The negative angle of the field is used for contrast estimation.

`nrefocus.metrics.metric_spectrum` (*rfi, distance, roi=None, **kwargs*)

Compute spectral contrast

Performs bandpass filtering in Fourier space according to optical limit of detection system, approximated by twice the wavelength.

`nrefocus.metrics.METRICS = {'average gradient': <function metric_average_gradient>, 'rms c`
Available metrics

3.3 Legacy methods

These methods are legacy functions which are kept for backwards-compatibility.

3.3.1 Refocusing

`nrefocus.refocus` (*field*, *d*, *nm*, *res*, *method*='helmholtz', *padding*=True)
Refocus a 1D or 2D field

Parameters

- **field** (*1d or 2d array*) – 1D or 2D background corrected electric field (Ex/BEx)
- **d** (*float*) – Distance to be propagated in pixels (negative for backwards)
- **nm** (*float*) – Refractive index of medium
- **res** (*float*) – Wavelength in pixels
- **method** (*str*) – Defines the method of propagation; one of
 - "helmholtz" : the optical transfer function $\exp(ik(M-1))$
 - "fresnel" : paraxial approximation $\exp(ik^2/k)$
- **padding** (*bool*) – perform padding with linear ramp from edge to average to reduce ringing artifacts.

New in version 0.1.4.

Returns

Return type Electric field at *d*.

`nrefocus.refocus_stack` (*fieldstack*, *d*, *nm*, *res*, *method*='helmholtz', *num_cpus*=2, *copy*=True, *padding*=True)
Refocus a stack of 1D or 2D fields

Parameters

- **fieldstack** (*2d or 3d array*) – Stack of 1D or 2D background corrected electric fields (Ex/BEx). The first axis iterates through the individual fields.
- **d** (*float*) – Distance to be propagated in pixels (negative for backwards)
- **nm** (*float*) – Refractive index of medium
- **res** (*float*) – Wavelength in pixels
- **method** (*str*) – Defines the method of propagation; one of
 - "helmholtz" : the optical transfer function $\exp(ik(M-1))$
 - "fresnel" : paraxial approximation $\exp(ik^2/k)$
- **num_cpus** (*int*) – Defines the number of CPUs to be used for refocusing.
- **copy** (*bool*) – If False, overwrites input stack.
- **padding** (*bool*) – Perform padding with linear ramp from edge to average to reduce ringing artifacts.

New in version 0.1.4.

Returns

Return type Electric field stack at d .

3.3.2 Autofocusing

`nrefocus.autofocus` (*field*, *nm*, *res*, *ival*, *roi=None*, *metric='average gradient'*, *padding=True*, *ret_d=False*, *ret_grad=False*, *num_cpus=1*)

Numerical autofocusing of a field using the Helmholtz equation.

Parameters

- **field** (*1d or 2d ndarray*) – Electric field is BG-Corrected, i.e. $\text{field} = \text{EX}/\text{BEx}$
- **nm** (*float*) – Refractive index of medium.
- **res** (*float*) – Size of wavelength in pixels.
- **ival** (*tuple of floats*) – Approximate interval to search for optimal focus in px.
- **roi** (*rectangular region of interest (x1, y1, x2, y2)*) – Region of interest of *field* for which the metric will be minimized. If not given, the entire *field* will be used.
- **metric** (*str*) –
 - “average gradient” : average gradient metric of amplitude
 - “rms contrast” : RMS contrast of phase data
 - “spectrum” : sum of filtered Fourier coefficients
- **padding** (*bool*) – Perform padding with linear ramp from edge to average to reduce ringing artifacts.
 Changed in version 0.1.4: improved padding value and padding location
- **ret_d** (*bool*) – Return the autofocusing distance in pixels. Defaults to False.
- **ret_grad** (*bool*) – Return the computed gradients as a list.
- **num_cpus** (*int*) – Not implemented.

Returns

- *field*, [*d*, [*grad*]]
- *The focused field and optionally, the optimal focusing distance and*
- *the computed gradients.*

`nrefocus.autofocus_stack` (*fieldstack*, *nm*, *res*, *ival*, *roi=None*, *metric='average gradient'*, *padding=True*, *same_dist=False*, *ret_ds=False*, *ret_grads=False*, *num_cpus=2*, *copy=True*)

Numerical autofocusing of a stack using the Helmholtz equation.

Parameters

- **fieldstack** (*2d or 3d ndarray*) – Electric field is BG-Corrected, i.e. $\text{Field} = \text{EX}/\text{BEx}$
- **nm** (*float*) – Refractive index of medium.
- **res** (*float*) – Size of wavelength in pixels.
- **ival** (*tuple of floats*) – Approximate interval to search for optimal focus in px.

- **roi** (*rectangular region of interest (x1, y1, x2, y2)*) – Region of interest of *field* for which the metric will be minimized. If not given, the entire *field* will be used.
- **metric** (*str*) – see *autofocus_field*.
- **padding** (*bool*) – Perform padding with linear ramp from edge to average to reduce ringing artifacts.

Changed in version 0.1.4: improved padding value and padding location

- **same_dist** (*bool*) – Refocus entire sinogram with one distance.
- **ret_ds** (*bool*) – Return the autofocusing distances in pixels. Defaults to False. If *same_dist* is True, still returns autofocusing distances of first pass. The used refocusing distance is the average.
- **ret_grads** (*bool*) – Return the computed gradients as a list.
- **num_cpus** (*int*) – Number of CPUs to use
- **copy** (*bool*) – If False, overwrites input array.

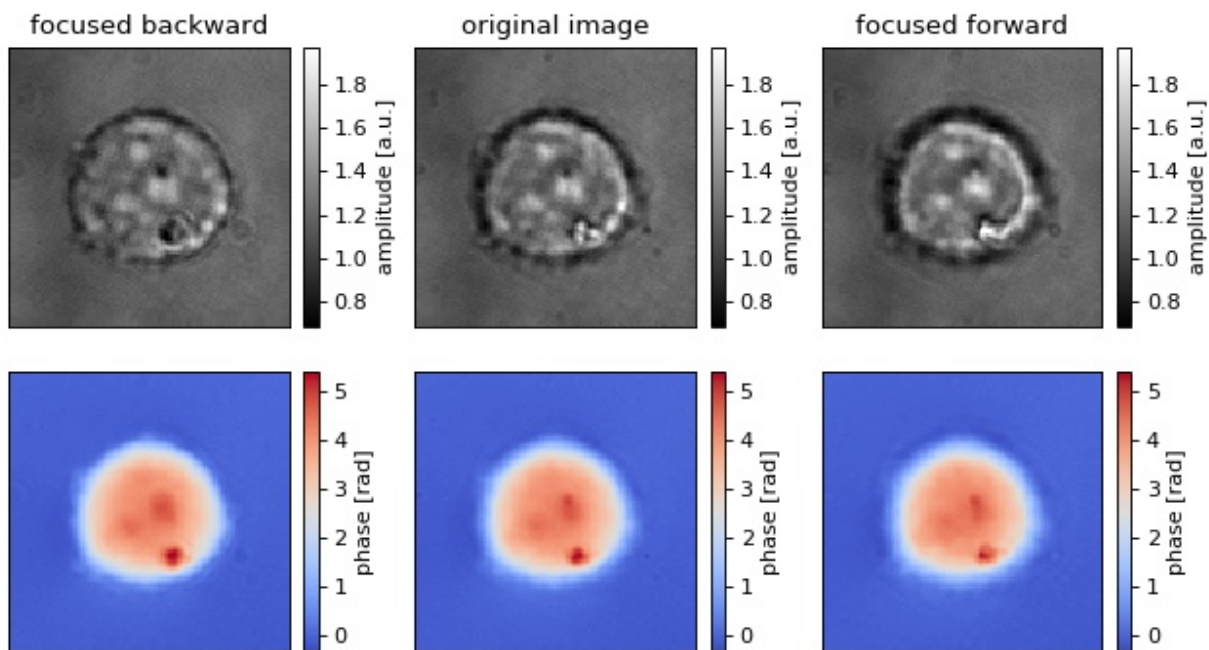
Returns

Return type The focused field (and the refocussing distance + data if *d* is None)

EXAMPLES

4.1 2D Refocusing of an HL60 cell

The data show a live HL60 cell imaged with quadriwave lateral shearing interferometry (SID4Bio, Phasics S.A., France). The diameter of the cell is about 20 μm .



refocus_cell.py

```
1 import matplotlib.pyplot as plt
2 import numpy as np
3 import unwrap
4
5 import nrefocus
6
7 from example_helper import load_cell
8
9 # load initial cell
10 cell1 = load_cell("HL60_field.zip")
```

(continues on next page)

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```

11
12 # refocus to two different positions
13 cell2 = nrefocus.refocus(cell1, 15, 1, 1) # forward
14 cell3 = nrefocus.refocus(cell1, -15, 1, 1) # backward
15
16 # amplitude range
17 vmina = np.min(np.abs(cell1))
18 vmaxa = np.max(np.abs(cell1))
19 ampkw = {"cmap": plt.get_cmap("gray"),
20          "vmin": vmina,
21          "vmax": vmaxa}
22
23 # phase range
24 cell1p = unwrap.unwrap(np.angle(cell1))
25 cell2p = unwrap.unwrap(np.angle(cell2))
26 cell3p = unwrap.unwrap(np.angle(cell3))
27 vminp = np.min(cell1p)
28 vmxp = np.max(cell1p)
29 phakw = {"cmap": plt.get_cmap("coolwarm"),
30          "vmin": vminp,
31          "vmax": vmxp}
32
33 # plots
34 fig, axes = plt.subplots(2, 3, figsize=(8, 4.5))
35 axes = axes.flatten()
36 for ax in axes:
37     ax.xaxis.set_major_locator(plt.NullLocator())
38     ax.yaxis.set_major_locator(plt.NullLocator())
39
40 # titles
41 axes[0].set_title("focused backward")
42 axes[1].set_title("original image")
43 axes[2].set_title("focused forward")
44
45 # data
46 mapamp = axes[0].imshow(np.abs(cell3), **ampkw)
47 axes[1].imshow(np.abs(cell1), **ampkw)
48 axes[2].imshow(np.abs(cell2), **ampkw)
49 mappha = axes[3].imshow(cell3p, **phakw)
50 axes[4].imshow(cell1p, **phakw)
51 axes[5].imshow(cell2p, **phakw)
52
53 # colobars
54 cbkwargs = {"fraction": 0.045}
55 plt.colorbar(mapamp, ax=axes[0], label="amplitude [a.u.]", **cbkwargs)
56 plt.colorbar(mapamp, ax=axes[1], label="amplitude [a.u.]", **cbkwargs)
57 plt.colorbar(mapamp, ax=axes[2], label="amplitude [a.u.]", **cbkwargs)
58 plt.colorbar(mappha, ax=axes[3], label="phase [rad]", **cbkwargs)
59 plt.colorbar(mappha, ax=axes[4], label="phase [rad]", **cbkwargs)
60 plt.colorbar(mappha, ax=axes[5], label="phase [rad]", **cbkwargs)
61
62 plt.tight_layout()
63 plt.show()

```

BILBLIOGRAPHY

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